

C. U. SHAH UNIVERSITY

Summer Examination-2022

Subject Name: Transform Methods

Subject Code: 4SC05TRM1

Branch: B.Sc. (Mathematics)

Semester: 5

Date: 28/04/2022

Time: 11:00 To 02:00

Marks: 70

Instructions:

- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

Q-1 Attempt the following questions: (14)

- a) State Dirichlet's condition for Fourier series. (02)
- b) State First Shifting Theorem for Laplace transforms. (02)
- c) Check whether given function is odd or even, (02)

$$f(x) = \begin{cases} -1, & -\pi < x < -\frac{\pi}{2} \\ 0, & -\frac{\pi}{2} < x < \frac{\pi}{2} \\ 1, & \frac{\pi}{2} < x < \pi \end{cases}$$

- d) If $F(\lambda)$ is Fourier transform of $f(x)$ then prove that (02)

$$F\{f(x) \cos ax\} = \frac{1}{2} \{F(\lambda - a) + F(\lambda + a)\}$$

- e) $Z(1) = \underline{\hspace{2cm}}$. (01)

(a) $\frac{z}{z+1}$ (b) $\frac{z}{z-1}$ (c) $\frac{z}{(z+1)^2}$ (d) None

- f) $Z\left(\frac{1}{n!}\right) = \underline{\hspace{2cm}}$. (01)

(a) e^z (b) $e^{\frac{1}{z}}$ (c) e (d) None

- g) $L(\sin at) = \underline{\hspace{2cm}}$. (01)

(a) $\frac{s}{s^2+a^2}$ (b) $\frac{s}{s^2-a^2}$ (c) $\frac{a}{s^2+a^2}$ (d) None

- h) $L^{-1}\left\{\frac{1}{\sqrt{s}}\right\} = \underline{\hspace{2cm}}$. (01)

(a) $\frac{1}{\sqrt{t}}$ (b) $\frac{1}{\sqrt{\pi t}}$ (c) $\frac{1}{t}$ (d) None

- i) The period of $\sin x$ is $\underline{\hspace{2cm}}$. (01)

(a) π (b) 2π (c) $\frac{\pi}{2}$ (d) None

- j) If $f(x)$ is even function then Fourier co-efficient b_n in Fourier series in the interval $(-l, l)$ is $\underline{\hspace{2cm}}$. (01)

(a) 1 (b) -1 (c) 0 (d) None



Attempt any four questions from Q-2 to Q-8

Q-2 Attempt all questions (14)

a) State and prove Euler's formula for Fourier series expansion of a function $f(x)$. (07)

b) Find the Fourier series of the function $f(x) = \begin{cases} -k & ; -\pi < x < 0 \\ k & ; 0 < x < \pi \end{cases}$ and (07)
also deduce that $1 - \left(\frac{1}{3}\right) + \left(\frac{1}{5}\right) - \left(\frac{1}{7}\right) + \dots = \frac{\pi}{4}$.

Q-3 Attempt all questions (14)

a) Find the Fourier series of $f(x) = x^2$ in the interval $0 < x < 2$. (07)

b) Find the half range cosine series of $f(x) = \pi - x, 0 < x < \pi$. (05)

c) Find Laplace transform of $f(t) = t \sin t$. (02)

Q-4 Attempt all questions (14)

a) Find Fourier transform of the function $f(x) = \begin{cases} 1 & , |x| < 1 \\ 0 & , |x| > 1 \end{cases}$ and hence (07)

evaluate $\int_0^\infty \frac{\sin \lambda}{\lambda} d\lambda$.

b) Using Fourier integral representation proves that (07)

$$\int_0^\infty \frac{\cos \lambda x + \lambda \sin \lambda x}{1 + \lambda^2} d\lambda = \begin{cases} 0 & \text{if } x < 0 \\ \frac{\pi}{2} & \text{if } x = 0 \\ \pi e^{-x} & \text{if } x > 0 \end{cases}.$$

Q-5 Attempt all questions (14)

a) State and prove Convolution Theorem for Laplace transform. (06)

b) Find inverse Laplace transform of $\frac{1}{s(s+1)(s+2)(s+3)}$. (06)

c) Find $L^{-1} \left\{ \frac{1}{s(s^2+a^2)} \right\}$. (02)

Q-6 Attempt all questions (14)

a) Find the Fourier sine and cosine transform of $f(x) = e^{-ax}, a > 0$. (05)

b) Find Laplace transform of the following, (05)

i) $e^{-3t}(\cos 4t + 3\sin 4t)$, ii) $2t^3 + e^{-2t} + t^{\frac{4}{3}}$

c) If $Z(u_n) = U(z)$ then prove that $Z(u_{n-k}) = z^{-k}U(z)$, where $k > 0$. (04)

Q-7 Attempt all questions (14)

a) Solve given differential equation by using Laplace transform. (07)

$$y'' - 10y' + 9y = 5t, y(0) = -1, y'(0) = 2$$

b) Find the Fourier cosine transform of $f(x) = \begin{cases} x, & 0 < x < 1 \\ 2 - x, & 1 < x < 2 \\ 0, & x > 2 \end{cases}$ (04)

c) If $F(\lambda)$ is fourier transforme of $f(x)$ then show that (03)
 $F(f(x - a)) = e^{ia\lambda} F(\lambda)$.



Q-8

Attempt all questions

(14)

- a) Prove that $Z(n^p) = -Z \frac{d}{dZ} (Z(n^p))$, where p is positive integer. (05)
- b) If $U(z) = \frac{2z^2+3z+12}{(z-1)^4}$ then find the value of u_1 and u_2 . (05)
- c) Find $Z(\cos n\theta)$. (04)

